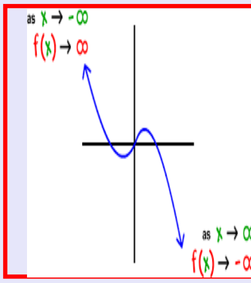
Math 245

Spring 2022

Lecture 44



Graph $f(x) = \dfrac{x^2 - x - 6}{x - 1}$

1) Domain: $x - 1 \neq 0$, $x \neq 1 \Rightarrow (-\infty, 1) \cup (1, \infty)$

2) Vertical Asymptote $x = 1$

Horizontal Asymptote None Deg. of num. > Deg. of deno.

Slant Asymptote Deg. of num. > Deg. of deno. by 1

Do long division

$$\begin{array}{r} x \\ x-1 \,\big)\, \overline{x^2 - x - 6} \\ -(x^2 - x) \\ \hline -6 \end{array}$$

$f(x) = x + \dfrac{-6}{x - 1}$

S.A. $\Rightarrow y = x$

Y-Int $(0, 6)$ x-Int $(3, 0)$, $(-2, 0)$

$x^2 - x - 6 = 0 \Rightarrow (x - 3)(x + 2) = 0$

Try few values

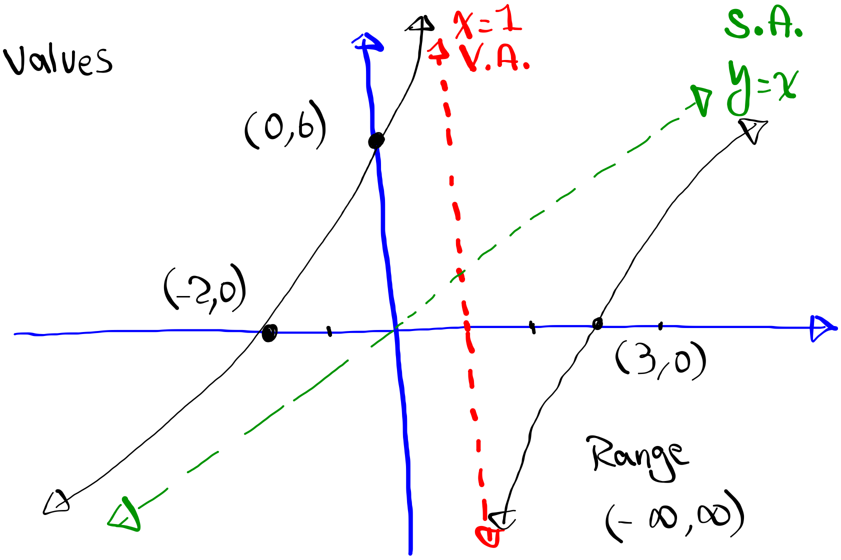
for x

$x=4 \to y$

$x=2 \to$

$x=-1 \to$

$x=-3 \to$



find

1) $_{12}C_5 = \dfrac{12!}{5!\cdot(12-5)!} = \dfrac{12!}{5!\cdot 7!} = \dfrac{\cancel{12}\cdot 11\cdot \cancel{10}\cdot 9\cdot 8\cdot \cancel{7!}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}\cdot\cancel{7!}}$

$= 11\cdot 9\cdot 8 = \boxed{792}$

2) $\dbinom{12}{7} = \dfrac{12!}{7!\cdot(12-7)!} = \dfrac{12!}{7!\cdot 5!} = \boxed{792}$

3) $_nC_0 = \dfrac{n!}{\cancel{0!}^{\,1}\cdot(n-0)!} = \dfrac{n!}{1\cdot n!} = \dfrac{n!}{n!} = \boxed{1}$

4) $_nC_n = 1$, $_nC_1 = n$

Expand $(a+b)^8$

1) 9 terms $(8+1)$

2) degree of each term $= 8$

$$1a^8b^0 + a^7b^1 + 28a^6b^2 + a^5b^3 + 70a^4b^4 + \cdots + 1a^0b^8$$

$$\binom{8}{2} \qquad \binom{8}{4}$$

Find the 4th term of $(x^2-y^4)^8$

$$(a+b)^8 \qquad a = x^2 \qquad b = -y^4$$

$$\binom{8}{3}a^5b^3 = 56(x^2)^5\cdot(-y^4)^3$$

$$= 56x^{10}\cdot(-y^{12})$$

$$= \boxed{-56x^{10}y^{12}}$$

Consider $(3x-4y^2)^{10}$

1) How many terms does this have?

$10+1$ terms $\Rightarrow$ 11 terms

2) Find the first 3 terms.

Expand $(a+b)^{10}$ where $a=3x$, and $b=-4y^2$

$\binom{10}{0}a^{10}b^{0} + \binom{10}{1}a^{9}b^{1} + \binom{10}{2}a^{8}b^{2} + \cdots$

$= (3x)^{10} + 10(3x)^9(-4y^2) + 45(3x)^8\cdot(-4y^2)^2 + \cdots$

$= 3^{10}x^{10} - 10\cdot 3^9\cdot 4\,x^9y^2 + 45\cdot 3^8\cdot 4^2\,x^8y^4$

Consider $(2x^3+1)^6$

1) How many terms?

$(6+1)$ terms $\Rightarrow$ 7 terms

2) Find the 5th term.

5th term of $(a+b)^6$ where $a=2x^3$, $b=1$

$\binom{6}{4}a^2b^4 = 15\cdot(2x^3)^2\cdot(1)^4$

$= 15\cdot 2^2\cdot(x^3)^2\cdot 1$

$= \boxed{60x^6}$

So the $(k+1)$th term of $(a+b)^n$ is

$$\binom{n}{k} a^{n-k} b^k$$

$n = 11$

Find 8 th term of $\left(2x - \frac{1}{2}\right)^{11}$.

$k+1 = 8 \rightarrow k = 7$ $\quad a = 2x \quad b = \frac{-1}{2}$

$$\binom{11}{7} a^{11-7} b^7 = \binom{11}{7} a^4 b^7 = 330 \cdot (2x)^4 \cdot \left(\frac{-1}{2}\right)^7$$

$$= 330 \cdot \cancel{2^4} \cdot x^4 \cdot \frac{-1}{\cancel{2^7}_{\,2^3}}$$

$$= -\frac{330}{8} x^4 = \boxed{-41.25x^4}$$

8th term of $\left(2x - \frac{1}{2}\right)^{11}$

Class QZ 10

1) Find $9! = \boxed{362\,880}$

2) Find $_{10}C_4 = \boxed{210}$

3) Find $\binom{10}{6} = \boxed{210}$